I

解答例

(1)

$$ {}^t(AB) = \begin{pmatrix} 9 & -10 & 8 \\ 7 & -4 & 39 \\ 23 & -16 & -14 \end{pmatrix} $$

(2)

$$ \begin{vmatrix} 4 & -3 & 0 \\ 3 & -1 & 1 \\ -2 & 2 & -2 \end{vmatrix} = -12 $$

(3)

$$ \begin{pmatrix} 3 & 3 & -1 \\ -2 & 1 & 1 \\ -3 & 3 & 1 \end{pmatrix}^{-1} = \frac{1}{6}\begin{pmatrix} 2 & 6 & -4 \\ 1 & 0 & 1 \\ 3 & 18 & -9 \end{pmatrix} $$

(4)

$$ (2-\lambda)(1-\lambda) - 3\cdot 2 = \lambda^2 - 3\lambda - 4 = (\lambda-4)(\lambda+1) = 0 $$

固有値は$\lambda=4,-1$

固有ベクトルを ${}^t(s_1, s_2)$とすると

$\lambda = 4$のとき

$$ \begin{pmatrix} -2 & 3 \\ 2 & -3 \end{pmatrix}\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} $$

$$ -2s_1 + 3s_2 = 0 $$

$$ \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = c\begin{pmatrix} 3 \\ 2 \end{pmatrix} \quad (c \neq 0) $$

$\lambda = -1$のとき

$$ \begin{pmatrix} 3 & 3 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} $$

$$ 3s_1 + 3s_2 = 0 $$

$$ \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = c\begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad (c \neq 0) $$

解答例

2

(1)

$y' = (3x^2 + 2)\cos(x^3 + 2x)$

(2)

$\frac{\partial z}{\partial x} = 2xy^2\cos(x^2y^2 + y)$

(3)

$\int\left(\cos\ x + \frac{2}{x^2}\right)dx = \sin x - \frac{2}{x} + C$ **（Cは積分定数）**

(4)

$\int_0^{2\pi}\sin^2 2x\,dx = \left[\frac{x}{2} - \frac{\sin 4x}{8}\right]_0^{2\pi} = (\pi - 0) - (0) = \pi$

解答例

3

複素数zが$z^6=1$を満たすとき、$ze^{i\frac{\pi}{3}}$を$x+yi$の形で求めよ。またそれらを複素平面上に図示せよ。

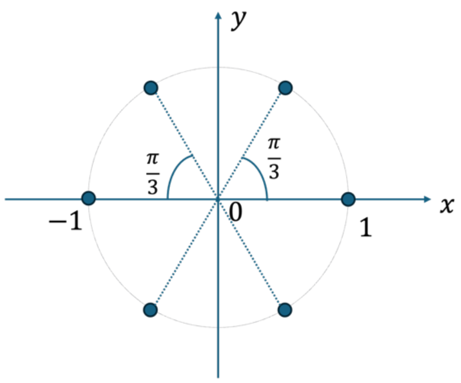
(5)　$z=re^{i\theta}$**とすると、**$z^6=r^6e^{6\theta i}=1$ **のため**$r=1$、$6\theta=2n\pi$ **5 点**

よって　$\theta=\frac{n\pi}{3}$　**これより、**$0\le\theta<2\pi$**の時、**$\theta=0,\frac{\pi}{3},\frac{2\pi}{3},\pi,\frac{4\pi}{3},\frac{5\pi}{3}$　**5 点**

よって

$$ze^{i\frac{\pi}{3}}=re^{i\theta}e^{i\frac{\pi}{3}}=e^{i(\theta+\frac{\pi}{3})}=e^{\frac{\pi}{3}i},e^{\frac{2\pi}{3}i},e^{\pi i},e^{\frac{4\pi}{3}i},e^{\frac{5\pi}{3}i},e^{2\pi i}$$

$$=\frac{1+\sqrt{3}i}{2},\frac{-1+\sqrt{3}i}{2},-1,\frac{-1-\sqrt{3}i}{2},\frac{1-\sqrt{3}i}{2},1$$



解答例

4

$$\nabla \cdot (\boldsymbol{E} \times \boldsymbol{H}) = \boldsymbol{H} \cdot (\nabla \times \boldsymbol{E}) - \boldsymbol{E} \cdot (\nabla \times \boldsymbol{H})$$

$$\begin{aligned}
\nabla \cdot (\boldsymbol{E} \times \boldsymbol{H}) &= \frac{\partial}{\partial x}(\boldsymbol{E} \times \boldsymbol{H})_x + \frac{\partial}{\partial y}(\boldsymbol{E} \times \boldsymbol{H})_y + \frac{\partial}{\partial z}(\boldsymbol{E} \times \boldsymbol{H})_z \\
&= \frac{\partial}{\partial x}\left(E_y H_z - E_z H_y\right) + \frac{\partial}{\partial y}(E_z H_x - E_x H_z) + \frac{\partial}{\partial z}\left(E_x H_y - E_y H_x\right) \\
&= \frac{\partial E_y}{\partial x} H_z - \frac{\partial E_z}{\partial x} H_y + E_y \frac{\partial H_z}{\partial x} - E_z \frac{\partial H_y}{\partial x} \\
&+ \frac{\partial E_z}{\partial y} H_x - \frac{\partial E_x}{\partial y} H_z + E_z \frac{\partial H_x}{\partial y} - E_x \frac{\partial H_z}{\partial y} \\
&+ \frac{\partial E_x}{\partial z} H_y - \frac{\partial E_y}{\partial z} H_x + E_x \frac{\partial H_y}{\partial z} - E_y \frac{\partial H_x}{\partial z} \\
&= H_x \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) + H_y \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) + H_z \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) \\
&- E_x \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) - E_y \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right) - E_z \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) \\
&= \boldsymbol{H} \cdot (\nabla \times \boldsymbol{E}) - \boldsymbol{E} \cdot (\nabla \times \boldsymbol{H})
\end{aligned}$$